

Top-level UDE

$$\frac{\partial r}{\partial u} \frac{du}{dr} = I = \frac{\partial r^T}{\partial u} \frac{du^T}{dr}$$

Sub-level total derivatives
= top-level partial derivatives

$$I = \frac{\partial r^T}{\partial u} \frac{du^T}{dr}$$

Sub-level UDE